Assignment 24

1. Let (M, d_M) and (N, d_N) be metric spaces. Show that
$$f \in \mathrm{C}(M, N) \iff f(\overline{A}) \subset \overline{f(A)}\,, \ A \subset M\,.$$
2. Let $f : (-1, 1) \to \mathbb{R}$ be such that
$$f(0) = 0 \text{ and } f(x) \geq c\,|x|^\alpha\,, \ x \in (-1, 1)$$
for some $\alpha \in (0, 1)$ and some $c > 0$. Conclude that f is not differentiable at $x = 0$.

3. Show that $\mathrm{O}(n) := \{M \in \mathbb{R}^{n\times n} \mid MM^T = \mathbf{1}_n\}$ is a compact subset of $\mathbb{R}^{n\times n}$.

4. Show that the series
$$\sum_{n=1}^{\infty} \bigl(1 - \cos(x/n)\bigr)$$
converges uniformly on compact subsets of $\mathbb{R}$.

The Homework is due Friday, May 30.