Assignment 23

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1. Let $(x_n)_{n\in\mathbb{N}} \in \mathbb{R}^{\mathbb{N}}$ and define $X = \{x_n \,|\, n \in \mathbb{N}\}$. Show by example that the limit points of X are accumulation points of $(x_n)_{n\in\mathbb{N}}$ but not vice-versa.

2. Let (M, d) be a metric space. Show that $A \cup B$ is compact whenever $A, B \subset M$ are.

3. Let $f \in \mathrm{C}(\mathbb{R}, \mathbb{R})$. Is it true that
$$f(\limsup_{n\to\infty} x_n) = \limsup_{n\to\infty} f(x_n) \text{ ?}$$

4. Let $f : D_f \to \mathbb{R}$ be differentiable at $x_0 \in D_f \overset{o}{\subset} \mathbb{R}$ and show that
$$\lim_{h\to 0} \frac{f(x+h) - f(x-h)}{2h} = f'(x_0)\,.$$

The Homework is due Friday, May 23.