# Assignment 15

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1. Let $\alpha \in (0,1)$ and assume that $A \subset \mathrm{C}^\alpha([0,1],\mathbb{K})$ be a bounded subset, that is, assume that
$$\|f\|_\alpha \leq M\,,\ f \in A$$
for some $M > 0$. Show that $A$ is uniformly equicontinuous.

2. Let $f, g \in \mathrm{C}_c(\mathbb{R},\mathbb{K})$ and show that
$$\begin{aligned}\operatorname{supp}(f * g) &\subset \operatorname{supp}(f) + \operatorname{supp}(g)\\ &=: \big\{x + y \,\big|\, x \in \operatorname{supp}(f)\,,\ y \in \operatorname{supp}(g)\big\}\,.\end{aligned}$$

3. Let $0 \leq f \in \mathrm{C}_c(\mathbb{R},\mathbb{R})$ with $\int_{-\infty}^{\infty} f(x)\,dx = 1$. Show that $(f_n)_{n\in\mathbb{N}}$ defined through
$$f_n(x) = n\,f(n\,x)\,,\ x \in \mathbb{R}\,,\ n \in \mathbb{N}\,,$$
is an approximate identity.

Let $(M, d)$ be a metric space. The set
$$\mathbb{B}(x,\varepsilon) = \{y \in M \,|\, d(x,y) < \varepsilon\}$$
is called "open" ball of radius $\varepsilon > 0$ about $x \in M$. A set $O \subset M$ is called open iff $\forall x \in O\ \exists\, \varepsilon > 0$ s.t. $\mathbb{B}(x,\varepsilon) \subset O$.

4. Let $(M, d)$ be a metric space and show that
$$\tau := \{O \subset M \,|\, O \text{ is open}\}$$
defines a topology on $M$. Compute $\tau$ for $M = \mathbb{R}$ and $d$ defined by
$$d(x,y) = \begin{cases} 0\,, & x = y\\ 1\,, & x \neq y\,.\end{cases}$$
For $M = \mathbb{R}$ define
$$d(x,y) = \frac{2}{\pi}\int_{\min(x,y)}^{\max(x,y)} \frac{d\xi}{1+\xi^2}\,,\ x, y \in \mathbb{R}\,.$$
Show that $(M, d)$ is a metric space and compute $\mathbb{B}(0,1)$, $\mathbb{B}(1,\frac{\pi}{4})$ .

5. You ask a question.

The Homework is due on Friday, March 7