Assignment 11

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1. Let $a < c < b \in \mathbb{R}$ and $f \in \mathcal{R}([a,b],\mathbb{R})$. Show that
$$f \in \mathcal{R}([a,c],\mathbb{R}) \cap \mathcal{R}([c,b],\mathbb{R}) \text{ and}$$
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx\,.$$
2. Let $a < b \in \mathbb{R}$ and $f \in \mathcal{R}([a,b],\mathbb{R})$. Define $F : [a,b] \to \mathbb{R}$ by
$$F(x) = \int_a^x f(y)\,dy\,,\ x \in [a,b]\,.$$
Prove that $F \in \mathrm{C}^{1-}\big([a,b],\mathbb{R}\big)$. When is F differentiable?

Let $(x_n)_{n\in\mathbb{N}}$ be a sequence in $\mathbb{R}^m$ for some $m \geq 1$. Then
$$x_n \underset{n\to\infty}{\longrightarrow} x_\infty \in \mathbb{R}^m :\iff$$
$$\Big[\sum_{k=1}^m (x_n^k - x_\infty^k)^2\Big]^{\frac{1}{2}} =: |x_n - x_\infty|_2 \underset{n\to\infty}{\longrightarrow} 0$$
A function $f : [a,b] \to \mathbb{R}^n$ is said to be Riemann-integrable, or concisely $f \in \mathcal{R}([a,b],\mathbb{R}^n)$ iff
$$\lim_{\triangle(P)\to 0} S(f,P) \in \mathbb{R}^n$$
exists for every Cauchy sum.

3. Show that
$$f \in \mathcal{R}([a,b],\mathbb{R}^n) \iff f^k \in \mathcal{R}([a,b],\mathbb{R})\,,\ k = 1,\dots,n$$
4. Let the function $f : [0,1] \to \mathbb{C}$ be given by
$$f(x) = \begin{cases} xe^{2i\pi/x}\,, & x \neq 0 \\ 0 & x = 0\,. \end{cases}$$
Show that $f \in \mathrm{C}([0,1],\mathbb{C})$ and plot $f\big([0,1]\big)$.

5. You ask a question.

The Homework is due on Friday January 31