Math 205 Winter Term 2006

Assignment 9

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1. Let $\alpha \in \mathrm{B}([a,b])$ be increasing and $f, g \in \mathcal{RS}\big([a,b]\big)$. Prove that $fg \in \mathcal{RS}\big([a,b]\big)$.

2. Let $\alpha \in \mathrm{B}([a,b])$ be increasing and $c \in (a,b)$. Assume that f and α are discontinuous from the right (or the left) at $x = c$ and prove that $\int_a^b f \, d\alpha$ cannot exist.

3. Let $\alpha \in \mathrm{B}([a,b])$ be increasing and compute
$$\int_a^b \mathbf{1}_{\{c,d\}} \, d\alpha$$
for $a \le c \le d \le b$ where $\{ = [\,,\ (\,$ and $\} =]\,,\ )$ and $\mathbf{1}_{\{c,d\}}$ is the characteristic function of the corresponding interval.

4. Assume that $f \in \mathrm{B}([a,b])$ is increasing and $\alpha \in \mathrm{C}([a,b])$. Show that there is $c \in [a,b]$ such that
$$\int_a^b f \, d\alpha = f(a) \int_a^c d\alpha + f(b) \int_c^b d\alpha \, .$$
[Hint: Mean Value Theorem.]

5. Prove the validity of Remarks 6.7.5 (c) and (d).

Homework due by Thurday, January 19 2006