Assignment 8

1. Show that the equation
$$e^{e^{\sin(x)}} = y$$
is uniquely solvable in a neighborhood U_0 of $x = 0$ for y in a neighborhood V_e of e. Can you compute an approximation to the exact solution x for $y \approx e$?

2. Let $k \in \mathbb{N}$ and show that
$$f : (0, \infty) \to \mathbb{R}\,,\ x \mapsto x^{1/k}$$
can be defined as the inverse of $g : (0, \infty) \to \mathbb{R}\,, x \mapsto x^k$ and compute f'.

3. Show that there exists a real real-valued continuously differentiable function which is invertible and for which the inverse is not Hölder continuous of any exponent.

4. Let $f \in \mathrm{C}^2\big((a, b), \mathbb{R}\big)$ and assume that $f''(x) \geq 0\,,\ x \in (a, b)$. Prove that
$$f\big(tx + (1 - t)y\big) \leq tf(x) + (1 - t)f(y)\,,\ t \in [0, 1]\,,\ x, y \in (a, b)\,.$$

5. Analyze the continuity/differentiability properties of the function f given by
$$f(x) = \frac{(x - 1)^2}{\log(x)}\,,\ x > 0\,.$$
Compute $f'(1)$ if it exists.

Homework due by Thursday, December 1 2005