Assignment 15

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1. Let $\alpha \in (0,1)$ and assume that $A \subset \mathrm{C}^\alpha([0,1],\mathbb{K})$ be a bounded subset, that is, assume that
$$\|f\|_\alpha = \|f\|_\infty + [f]_\alpha \le M\,,\ f \in A$$
for some $M > 0$. Show that A is uniformly equicontinuous.

2. Let $f, g \in \mathrm{C}_c(\mathbb{R},\mathbb{K})$ and show that
$$\begin{aligned}\operatorname{supp}(f*g) &\subset \operatorname{supp}(f) + \operatorname{supp}(g)\\ &=: \bigl\{x+y \,\big|\, x \in \operatorname{supp}(f)\,,\ y \in \operatorname{supp}(g)\bigr\}\,.\end{aligned}$$

3. Let $0 \le f \in \mathrm{C}_c(\mathbb{R},\mathbb{R})$ with $\int_{-\infty}^{\infty} f(x)\,dx = 1$. Show that $(f_n)_{n\in\mathbb{N}}$ defined through
$$f_n(x) = n\,f(n\,x)\,,\ x \in \mathbb{R}\,,\ n \in \mathbb{N}\,,$$
is an approximate identity.

4. Let $a < b \in \mathbb{R}$ and $\varepsilon > 0$ be given. Show that the constant function with value 1 on $[a,b]$ can be extended to a C^∞-function of the line which vanishes outside $[a-\varepsilon, b+\varepsilon]$.

5. Give an example of a uniformly bounded and uniformly equicontinuous sequence of functions on the whole real line which does not possess a uniformly convergent subsequence.

Homework due by Thursday, March 2 2006