Assignment 14

---

1. Let the function $f \in \mathrm{C}(\mathbb{R}, \mathbb{R})$ be given by
$$f(x) = \begin{cases} 1 - |x|\,, & |x| < 1\,, \\ 0\,, & |x| \geq 1\,. \end{cases}$$
Compute and plot $f * f$ and $f * f * f$.

2. Show that $f : \mathbb{R} \to \mathbb{R}\,,\ x \mapsto \frac{1}{1+x^2}$ is analytic.

3. Let $f \in \mathrm{C}_c(\mathbb{R}, \mathbb{R})$ and $g \in \mathrm{C}^1(\mathbb{R}, \mathbb{R})$. Show that
$$f * g \in \mathrm{C}^1(\mathbb{R}, \mathbb{R})\,.$$

4. For $f \in \mathrm{C}_c(\mathbb{R}, \mathbb{R})$ define its Fourier transform $\hat{f} : \mathbb{R} \to \mathbb{K}$ by
$$\hat{f}(\xi) = \int_{-\infty}^{\infty} e^{-ix\xi} f(x)\, dx\,.$$
Show that $\hat{f}$ is well-defined and analytic. Give an estimate for the radius of convergence of its power series expansion about $\xi = 0$.

5. Let $f \in \mathrm{C}^\omega\big((a,b), \mathbb{K}\big)$ and $c \in (a,b)$. Prove that F, given by
$$F(x) := \int_c^x f(y)\, dy\,,\ x \in (a,b)\,,$$
is also analytic.

Homework due by Thursday, February 23 2006