Assignment 4

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1. Let $A = (a_{jk})_{j,k\in\mathbb{N}}$ be a double array of real numbers and let
$$d = (d_1, d_2, d_3, \cdots) = (a_{11}, a_{21}, a_{12}, a_{31}, a_{22}, a_{13}, \cdots)$$
be the sequence obtained by concatenating the finite diagonals
$$d_m = (a_{m1}, a_{m-1\,2}, \cdots, a_{1m})\,,\ m \in \mathbb{N}\,.$$
Show that any limit point of any row $A_{j\bullet} = (a_{jk})_{k\in\mathbb{N}}$ or column $A_{\bullet k} = (a_{jk})_{j\in\mathbb{N}}$ of A is a also a limit point of the sequence d. Do we obtain all limit points of d this way?

2. Let $A \subset \mathbb{R}$. We say that $B \subset A$ is open in A, or, concisely $B \overset{o}{\subset} A$, iff there is an open set $\tilde{B} \subset \mathbb{R}$ with
$$B = A \cap \tilde{B}\,.$$
Show that
(i) $\emptyset, A \overset{o}{\subset} A$.
(ii) If $B_j \overset{o}{\subset} A$ for $j \in \mathbb{N}$, then $\bigcup_{j\in\mathbb{N}} B_j \overset{o}{\subset} A$.
(iii) If $B_j \overset{o}{\subset} A$ for $j = 1, \ldots, m$ $(m \in \mathbb{N})$, then $\bigcap_{1\le j\le m} B_j \overset{o}{\subset} A$.
Is $[0, 1/2)$ open in $[0, 1]$? What about $(0, 1/2]$? Is $\{0\}$ open in $\mathbb{N}$? Is it open in $\mathbb{Q}$?

3. Let $x \in \mathbb{R}^{\mathbb{N}}$ and let
$$X = \{y \in \mathbb{R} \,|\, y = x_j \text{ for some } j \in \mathbb{N}\}\,.$$
What is the relation between the limit points of the sequence x and those of the set X?

4. Let $A \subset \mathbb{R}$. The sets $\overline{A}$, $\overset{\circ}{A}$ and $LP(A)$ were defined in class. Let, in addition, $\partial A = \overline{A} \setminus \overset{\circ}{A}$. Prove or disprove the following:
$$\overset{\circ}{A} \subset \overline{A}\,,\ \overline{A} = LP(A)\,,$$
$$LP(A) \subset A\,,\ LP\big(LP(A)\big) \subset LP(A)\,,$$
$$LP(A) \subset LP\big(LP(A)\big)\,,\ \overline{\partial A} = \partial A\,,$$
$$\overline{A} = LP(A) \cup A\,,\ \partial(\partial A) = \partial A\,.$$

5. Let $x, y \in \mathbb{R}^{\mathbb{N}}$ be two sequences. Show that
$$\limsup_{k\to\infty}(x_k + y_k) \le \limsup_{k\to\infty} x_k + \limsup_{k\to\infty} y_k\,.$$

Homework due by Thursday, October 27 2005.