

Assignment 2

1. Let $(x_n)_{n \in \mathbb{N}} \in CS(\mathbb{Q})$ and consider the polynomial function defined through

$$p(x) = \sum_{j=1}^m p_j x^j, \quad x \in \mathbb{Q},$$

with rational coefficients $p_j \in \mathbb{Q}$, $j = 1, \dots, m$. Prove that the sequence $(p(x_n))_{n \in \mathbb{N}} \in CS(\mathbb{Q})$, too.

2. Describe the set $CS(\mathbb{N})$ and determine whether it is countable or not. [Hint: Prove first that the countable union of countable sets is countable.]
3. Prove that \mathbb{R} is uncountable and has the same cardinality as $2^{\mathbb{N}}$.
4. Prove that
$$|x - y| \geq ||x| - |y|| \quad \forall x, y \in \mathbb{R}.$$
5. Show that convergent sequences are Cauchy.