Assignment 1

1. Form the negation of the following statements:
(i) $\forall x \in A\, \exists\, y \in B : (x, y) \in G$.
(ii) $\forall x \in A\, \exists!\, y \in B : (x, y) \in G$.
(iii) $\exists\, y \in B : (x, y) \notin G\, \forall x \in A$.
(iv) $\forall\, n \in \mathbb{N}\, \exists\, m \in \mathbb{N} : m \geq n$ and m is prime.

2. Let A, B be non-empty sets. Then $G \subset A \times B$ determines a map
$$f_G : A \to B\,,\; x \mapsto y$$
iff
$$(x, y) \in G\,,\; (x, \tilde{y}) \in G \Longrightarrow y = \tilde{y}\,.$$
If this is the case, we define
$$\mathrm{dom}(f) := \{x \in A \,|\, \exists y \in B \text{ with } (x, y) \in G\}, \text{ the domain of } f,$$
$$\mathrm{im}(f) := \{y \in B \,|\, \exists x \in A \text{ with } (x, y) \in G\}, \text{ the range of } f.$$
Use quantifiers to formulate the following:
(i) The map is one-to-one.
(ii) The map is onto.
(iii) The map is bijective.
(iv) Given $\tilde{G} := \{(y, x) \in B \times A \,|\, (x, y) \in G\}$, determine what $f_{\tilde{G}}$ is, if it exists.

3. Let $\mathbb{N}_m := \{1\,, 2\,, \ldots\,, m\}$. How many maps $f : \mathbb{N}_m \to \mathbb{N}_n$ are there for $m, n \in \mathbb{N}$? How many are the bijections among them?
[Consider the cases $m < n$, $m = n$ and $m > n$, separately.]

4. Is the set
$$\mathrm{Map}(\mathbb{N}, \mathbb{N}) := \{f : \mathbb{N} \to \mathbb{N} \,|\, f \text{ is a map}\}$$
countable? What about
$$\mathrm{Map}_b(\mathbb{N}, \mathbb{N}) := \{f : \mathbb{N} \to \mathbb{N} \,|\, f \text{ is bijective}\}?$$
Justify your answers.

5. Consider the set of all *finite* subsets of $\mathbb{N}$. Is it countable?

Homework due by Tuesday, October 4 2005