to write simple and efficient programs to handle large problems. Ease of use does not lead to constriction of usage. Almost every example in this book shows how sophisticated mathematics can become a few lines of MATLAB.

REFERENCES

[1] The Mathworks Inc., *Using MATLAB*, available from www.mathworks.com, 6445K (pdf), November 2000.

[2] The Mathworks Inc., *Getting started with MATLAB*, available from www.mathworks.com, 2437K (pdf), September 2000.

Divakar Viswanath
University of Chicago

The Nonlinear Theory of Elastic Shells. Second Edition. *By A. Libai and J. G. Simmonds.* Cambridge University Press, Cambridge, UK, 1998. $85.00. xvi+542 pp., hardcover. ISBN 0-521-47236-9.

This book, authored by two of the foremost shell theorists of our time, is an expanded second edition of a book [1] published by Academic Press 10 years earlier.

In this expanded second edition, the authors have added to the original six chapters two new chapters on general nonlinear membrane theory and on general nonlinear shell theory. The book's original subtitle, *One Spatial Dimension*, is dropped because these general theories allow for variations of shell deformation in the directions of both middle surface curvilinear coordinates. The authors note that they have updated many parts of the first six chapters of the first edition, adding references and rewriting several sections to improve both their style and substance. The main modifications (other than correcting typos and adding new references) seem to consist of the following:

(1) A validation of the weak form of the equation of motion for rods with a doubly symmetric cross section (birods) in the presence of shocks.

(2) A different development of the principle of virtual work for birods, cylindrical motion of cylindrical shells (beamshells), and torsionless axisymmetric motion of shells of revolution (axishells).

(3) Some added paragraphs on the nonlinear, large-strain axisymmetric elastostatic theory of membranes to include new results that have appeared since the publication of the earlier edition of the book such as (i) large deformation problems in cylindrical and spherical inflatables and (ii) tension fields.

(4) A more extensive citation and description of existing results on the elastodynamics of axishells.

(5) A summary of a new dynamic theory of axishells, subject to the Kirchhoff–Love hypotheses, undergoing combined axisymmetric bending and torsional deformation, by one of the authors. (A special case involving infinitesimal strain/finite rotation elastostatics of shells of revolution was the subject of a Ph.D. dissertation by W. A. Smith in the mid-1960s. Regrettably, his results have not been published in an archival journal and hence are not readily accessible.)

(6) A new presentation of the bending strains of helicoidal shells subject to external loading that induces rotationally symmetric stresses and strains in the helicoid, stimulated by an observation in a recent dissertation on the nonuniqueness of the rotator tensor decomposition. (The rotator is the proper orthogonal tensor associated with the rigid body rotation that precedes the distortion part of the shell deformation.)

(7) A modified presentation of the compatibility conditions for rotationally symmetric strains of helicoidal shells.

As far as this reviewer can tell, there are no other substantive changes in the first six chapters from the earlier edition. As such, the original outstanding features of the first edition remain intact. Since the two new chapters follow nearly the same format as the earlier chapters, it seems appropriate to mention below the general approach taken by the authors to formulate shell theory and the particular way they have chosen to present their material throughout the book.

The book begins with a short first chapter describing the authors' view on shells and shell theory and a second chapter summarizing the basic theory of the mechanics of continuous media. Each succeeding chapter begins with an integral form of the equations of motion for dynamical variables that are functions of fewer spatial coordinates (than three) that are appropriate for the class of problems to be discussed (from birods to axishells and, in the new edition, to general deformable shells). These are exact consequences of the corresponding equations for three-dimensional continuum mechanics of Chapter II after suitable averaging (integration) to take advantage of the particular geometrical features such as slenderness or thinness and/or a particular type of deformations. Differential equations of motion involving fewer than three spatial dimensions typically follow from applications of some version of the divergence theorem. Jump conditions and propagation of singularities are discussed, leading to a weak form of the equations of motion.

The kinematics quantities such as strain and strain rates that characterize the deformation of the shell are then introduced in a natural way through the deformation power in the mechanical work identity. For problems in elastostatics, boundary conditions are formulated consistent with the principle of virtual work for the class of problems. A purely mechanical theory is then developed with the introduction of a strain energy density. Variational principles are formulated to provide an internal check for the consistency of the theory and to serve as a tool for approximate solution via the direct method of the calculus of variations. The early chapters typically end with a discussion of the thermodynamics of the particular class of problems and the related stability issues.

The approach to shell theory adopted by the authors is significant for a number of reasons. To this reviewer, the most important reason is that all the approximations needed to attain a particular type of shell theory now occur only in the formulation of the relevant constitutive laws (the stress-strain relations describing the material properties of the shell and, for nonisothermal deformation, the first law of thermodynamics, which involves the energy conservation for the shell). The dynamics and kinematics parts of the relevant shell theory remain exact consequences of the basic axioms of three-dimensional continuum mechanics unless we choose to take advantage of the special features of the bodies or the particular class of deformation (such as shallowness) to approximate and simplify these two groups of equations. It is intriguing that an approximation such as the Kirchhoff–Love hypothesis that has been traditionally viewed as an assumption that approximates the deformation of the shell appears as an approximation of the material properties of the shell in the approach taken by the authors.

In laying out the material of both the first and second editions of the book, the authors chose not to present the general theory first and then specialize it to the various special theories such as cylindrical bending, axisymmetric torsionless deformation of shells of revolution, membrane theory, etc. To them, "the best path to understanding general shell theory takes the reader upward through stations of increasing complexity, each station offering a view of interest in its own right." From the basic theory of three-dimensional continuum mechanics of Chapter II, the book first derives the one-dimensional longitudinal equation of motion for a rod of a doubly symmetric cross section (birod for short) in Chapter III. It is followed by a chapter (IV) on cylindrical motion of cylindrical shells with no deformation in the direction of the generators of the cylindrical middle surface (beamshells for short). The next two chapters (V and VI) are also devoted to developments of shell theories involving only one spatial coordinate. For axisymmetric deformation of shells of revolution (axishell), we now encounter the effects of middle surface curvature in two directions. The unishell theory in Chapter VI is a spatially one-dimensional theory only for quantities intrinsic to the shell such as stresses and strains, while the associated displacement and rotation components may vary in both middle surface coordinate directions.

The ascending approach to shell theory allows the authors "to elaborate on a concept just once in the simplest context. Thus, shock relations, mechanical boundary conditions, the Virtual Work Principle, variational formulations, and some basic techniques from thermodynamics are motivated and developed in the chapters on birods, while an extended discussion on load potentials, nonlinear stress-strain relations, and thermodynamic stability is to be found in the chapter on beamshells." This approach made it possible not to allocate pages in the last three chapters to the thermodynamic aspects (of the theories being discussed) similar to those already presented in the earlier chapters.

To this reviewer, the book provides readers with a good understanding of the general theoretical foundation of the different classes of shell theories. A great deal of care has been taken to clarify the relation between the approximate two-dimensional theories and the exact three-dimensional theory of the mechanics of continuous media. At the same time, specific problems are solved in some detail throughout one chapter to illustrate applications of the theories developed. The only exception is Chapter VIII on the general mechanical theory of shells. This may be explained by the fact that most solved problems suitable for presentation in this book have already been included in the earlier chapters in the ascending approach. The few that are truly spatially two-dimensional were already discussed in Chapter VII on nonlinear membrane theory. Other spatially two-dimensional problems are just too complex to permit any meaningful analytical solutions, especially when both geometrical nonlinearity and material nonlinearity are present.

The prevalence of analytically intractable two-dimensional shell problems is precisely what makes a book that emphasizes the foundation of the various types of shell theories that much more valuable. When it is necessary to seek approximate solutions to problems of interest via the discrete analogues of the continuum models and numerical techniques such as the finite element method, it is that much more important for the users of a shell theory to have a good grasp of the theory. But the authors do more than provide the users with a sound general shell theory. From their own research, the authors explore alternative formulations to a theory resulting from a straightforward descent from the three-dimensional continuum mechanics as well as simplify further the equations of well-established shell theories. This is another unique feature that could only be accomplished by a pair of scholars who have been successful in this type of research.

Regarding nonlinear membrane theory, it should be noted that the authors have made an unusual effort to summarize some very valuable but often overlooked research results of the last two decades on membrane wrinkling, tension fields, and membrane dynamics, in addition to synthesizing the more classical results. More generally, the wealth of solved problems involving both geometric and material nonlinearity reflects the authors' tremendous command of shell theory. The generality and clarity of the book's substantial treatment of material nonlinearity also makes it valuable and appealing to a broader readership, extending beyond engineers and mechanicians working in the more conventional areas of aerospace, mechanical, and structural engineering to include biomechanicians and mathematical physiologists.

While there is a rich collection of solutions for interesting nonlinear shell problems on the whole, the book is not for readers seeking a nonlinear shell theory counterpart of *Stresses in Shells* by W. Flugge or *The Theory of Plates and Shells* by S. Timoshenko and S. Woinowsky-Krieger. In light of the complexity of the subject matter, practitioners in the field of nonlinear shell theory need a book that sets up the theoretical framework clearly and formulates the relevant initial-boundary value problems correctly so that those who wish to obtain approximate solutions can do so properly and with a good understanding of the theoretical underpinning. Libai and Simmonds have offered us a monograph that uniquely meets this need. (They also demand from the reader a working knowledge of general tensor analysis, particularly for the two new chapters added in the second edition.) The virtuosity displayed by the authors in combining a clear exposition of the different shell theories with their masterful use of special problems to illustrate the applications of these theories is simply awesome to this reviewer, who is not exactly a neophyte in the field of plates and shells.

REFERENCE

[1] A. LIBAI AND J. G. SIMMONDS, *The Nonlinear Theory of Elastic Shells, One Spatial Dimension*, Academic Press, San Diego, London, 1988.

FREDERIC Y. M. WAN
University of California, Irvine

Statistics of Random Processes I: General Theory. *By R. S. Liptser and A. N. Shiryayev.* Springer-Verlag, New York, 2000. $72.00. xv+427 pp., hardcover. ISBN 3-540-63929-1.

Statistics of Random Processes II: Applications. *By R. S. Liptser and A. N. Shiryayev.* Springer-Verlag, New York, 2000. $72.00. xv+402 pp., hardcover. ISBN 3-540-63928-4.

Written by two renowned experts in the field, the books under review contain a thorough and insightful treatment of the fundamental underpinnings of various aspects of stochastic processes as well as a wide range of applications. Providing clear exposition, deep mathematical results, and superb technical presentation, they are masterpieces of the subject of stochastic analysis and nonlinear filtering. These books are the second edition of the two volumes whose original Russian edition *Statistika Sluchainyk Protsessov* (published by Nauka in Moscow, 1974) was translated by A.B. Aries in 1978 and published in the Springer-Verlag series "Applications of Mathematics."

What is special about these books is their broad coverage and in-depth study of optimal filtering problems. Before proceeding further, let us briefly mention the filtering-problem formulation. Let $(\Omega, \mathcal{F}, P)$ be the underlying probability space. Consider a partially observed process $\{(x_t, y_t)\}$, where $\{x_t\}$ is the state and $\{y_t\}$ is the observation. A nonlinear filtering problem can be stated as: At any time t, based on the observations $\{y_s : s \leq t\}$, one wishes to estimate the unobservable state x_t. If $E|x_t|^2 < \infty$, the mean square estimate $\widehat{x}_t = E(x_t|\mathcal{F}_t^y)$ is an optimal solution, where $\mathcal{F}_t^y$ denotes the σ-algebra generated by the observations $\{y_s : s \leq t\}$. Although in principle $\widehat{x}_t$ can be computed via the Bayes formula, even in simple cases, the computation involved could be rather cumbersome. As a result, such nonlinear filtering problems have been the focal point of an enormous research effort over the past 40 years since the original work of Kalman [2] and Kalman and Bucy [3] on linear filtering problems; see [1, 4, 5] for some of the references and developments. In the books under review, assuming that the pair (x_t, y_t) is a diffusion process with

(1)
$$\begin{aligned} dx_t &= a_1(x_t, y_t)dt + \sigma_1(x_t, y_t)dw_1(t), \\ dy_t &= a_2(x_t, y_t)dt + \sigma_2(x_t, y_t)dw_2(t), \end{aligned}$$

for appropriate functions $a_i(\cdot)$ and $\sigma_i(\cdot)$ and independent Wiener processes $w_1(\cdot)$ and $w_2(\cdot)$, the study of optimal filtering problems is then carried out.

Volume I begins with the essentials of probability theory by reviewing some main concepts from probability theory and stochastic processes including stopping times, Brownian motions, likelihood functions, efficiency, and Bayesian statistics, among others. The next two chapters are devoted to an important concept, martingales. The authors begin with discrete-time processes and extend the discussion to continuous-time cases. Stochastic integrals, the Itô formula, and solutions of stochastic differential equations are then presented. They proceed with the study of structural properties of functionals of Wiener processes, nonnegative supermartingales and martingales, and the Girsanov transformation. Then, another chapter concentrates on absolute continuity of measures. Among other topics, representation of Itô processes as diffusion processes, innovation processes, and the Cameron–Martin formula are discussed in detail. Next, the authors consider general filtering problems and derive the filtering equations, deal with optimal filtering, interpolation, and extrapolation of Markov processes with countable state space, and present the Kalman–Bucy filters.

Volume II is devoted to applications. It begins with the study of conditional Gaussian processes, examines optimal filtering of conditional Gaussian processes, and studies applications arising in statistics including such problems as maximum likelihood