Ch 1 Experimental Facts

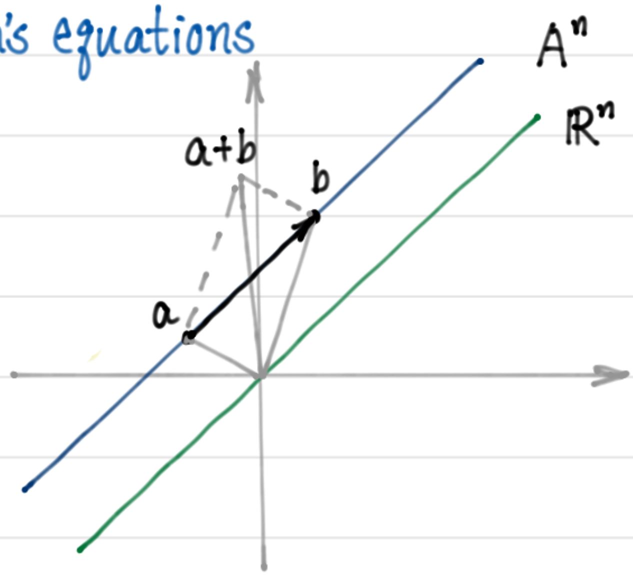
1 The principles of relativity and determinacy

A. Space and time: $\mathbb{R}^3 \times \mathbb{R}$

B. Galileo's principle of relativity: inertial coordinate systems

C. Newton's principle of determinacy: The initial location and velocity uniquely determines all of its motion.

2 The Galilean group and Newton's equations



$\mathbb{R}^n$ v.s. A^n

↑ no linear structure

A^n: no fixed origin

$a, b \in A^n$, $a+b$ not defined

$b-a \in \mathbb{R}^n$

$\rho(x,y) = \|x-y\| = (x-y, x-y)^{\frac{1}{2}}$ is well defined

E^n: $A^n + \rho(\cdot,\cdot)$ Euclidean space.

Galilean structure

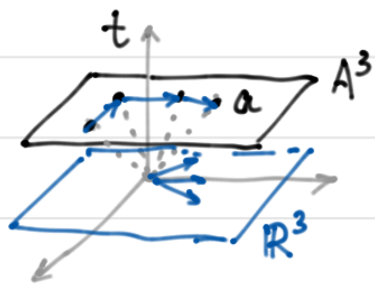
① A^4: the universe

$\mathbb{R}^4$: $b-a$ parallel displacement

② $t : \mathbb{R}^4 \to \mathbb{R}$ $\quad t(b-a)$, here $t(\cdot)$ is a mapping not multiplication

If $t(b-a)=0$, then events a and b are called simultaneous.

- $\mathrm{Ker}(t) = \mathbb{R}^3$ parallel displacements of simultaneous events
- Given a, $A^3(a)$: affine space of simultaneous (to a) events



③ A^3: $\rho(a,b) = \|a-b\| = (a-b, a-b)^{\frac{1}{2}}$ $a, b \in A^3$

$(\cdot,\cdot)$ is defined on $\mathbb{R}^3$.

$A^3 + \rho(\cdot,\cdot) \to E^3$.

- Galiean space: A^4 + ① ② ③ No coordinate chosen yet!
- Galiean group = {Galiean transformation}

affine transformation of A^4 which preserve $t(\cdot)$ and $\rho(\cdot,\cdot)$

Example. Galilean coordinate space $\mathbb{R} \times \mathbb{R}^3$ a special example of A^4

Galilean transformation. Dimension is 10.

① uniform motion with velocity v: $g_1(t,x) = (t, x+vt)$

② translation of the origin: $g_2(t,x) = (t+t_0, x+x_0)$

③ rotation of the coordinate: $g_3(t,x) = (t, Gx)$ $G: \mathbb{R}^3 \to \mathbb{R}^3$ orth

Motion, velocity, acceleration

motion $\vec{x}: I \to \mathbb{R}^N$, where $I \subset \mathbb{R}^1$ is an interval.

velocity: $\dot{x}(t_0)$

acceleration: $\ddot{x}(t_0)$

trajectory or curve: the image of a motion.

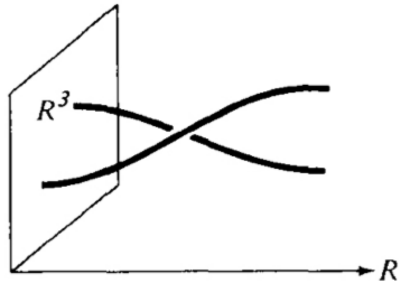


Figure 4 World lines

A motion of a system of n points. Let $x_i: \mathbb{R} \to \mathbb{R}^3$, $i = 1, \cdots, n$.

$\vec{x}: \mathbb{R} \to \mathbb{R}^N$, $N = 3n$

configuration space

Newton's equations $\quad \ddot{x} = F(x, \ddot{x}, t) \quad (1)$

Newton's principle of determinacy: The initial location and velocity uniquely determines all of its motion.

By ODE theory, $\vec{x}(t)$ is uniquely determined by F, $x(t_0)$, and $\dot{x}(t_0)$.

Constraints imposed by the Galileo's principle of relativity

Equation (1) must be invariant with respect to the group of Galilean transformations.

① time translation invariance. If $x = \varphi(t)$ solves (1), so is $x = \varphi(t+s)$ $\forall s \in \mathbb{R}$.

The laws of nature remain constant.

So $\ddot{x} = \Phi(x, \dot{x})$ this is called autonomous

② space translation invariance. If $x_i = \varphi_i(t)$ solves (1), so is $\varphi_i(t) + \vec{r}$, $\forall \vec{r} \in \mathbb{R}^3$.

relative coordinate

$x + vt$. $\dot{x} \to \dot{x} + v$ for a fixed $\vec{v}$. relative velocity

So $\ddot{x}_i = f_i(\{\underline{x_j - x_k}, \underline{\dot{x}_j - \dot{x}_k}\})$, $i, j, k = 1, \ldots, n$.

③ space rotation invariance. If φ_i solves (1), then so is $G\varphi_i$.

$G: \mathbb{R}^3 \to \mathbb{R}^3$ orthogonal transformation. space is isotropic.

So $F(Gx, G\dot{x}) = GF(x, \dot{x})$

Examples. Weight on a spring.

$$\ddot{x} = -\alpha^2 x$$

$$U(x) = \frac{1}{2}\alpha^2 x^2$$

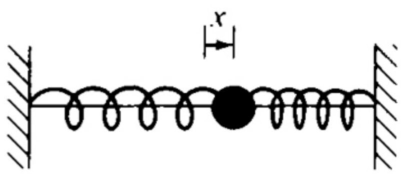


Figure 8 Weight on a spring

Examples. Conservative systems

$$m_i \ddot{x}_i = -\frac{\partial U}{\partial x_i}, \quad i = 1, \ldots, n.$$

$U: E^{3n} \to \mathbb{R}$ the poential field.