PROJECT 1: HEAT EQUATION

We shall solve the heat equation

$$\begin{aligned} u_t &= \Delta u + f && \text{in } \Omega \times (0,T) \\ u &= g && \text{on } \partial\Omega \times (0,T) \\ u(x,0) &= u_0(x) && \text{in } \Omega. \end{aligned}$$

(1) Code forward Euler, backward Euler, and Crack-Nicolson method for 2-D heat equation on the unite square (you are free to choose finite difference or finite element method).

(2) Check the convergence rate in time and space using the exact solution

$$u(x,t) = \beta(t) * e^{-[(x-t+0.5)^2+(y-t+0.5)^2]/0.04}$$

with

$$\beta(t) = 0.1 * (1 - e^{-10^2*(t-0.5)^2}).$$

Hint: To check the rate in time, you can fix a small h in space and let Δt vary and vice verse.

(3) Using your algorithm to create a movie for the above solution evolving from $t = 0$ to 1.