

HW2 OF MATH 226 B

LONG CHEN

Due date: Feb 4

- (1) Construct an example to show that if $\Delta t > h^2/2$, then

$$\|\mathbf{U}^n\|_\infty > \|\mathbf{U}^0\|_\infty.$$

- (2) Apply backward Euler to the example you constructed in (1) to show numerically the scheme is stable.
- (3) Prove the maximum norm stability of Crank-Nicolson method. What is the weakest assumption on λ you need? Is it necessary?
- (4) (The θ method). For $\theta \in [0, 1]$, we use the scheme

$$(1) \quad \frac{U_i^{n+1} - U_i^n}{\Delta t} = (1 - \theta) \frac{U_{i-1}^n + U_{i+1}^n - 2U_i^n}{h^2} + \theta \frac{U_{i-1}^{n+1} + U_{i+1}^{n+1} - 2U_i^{n+1}}{h^2}.$$

Give a complete error analysis (consistency, stability, and convergence) of the θ method.