

HW2 of Math 226C

Due May 18

1. Prove that if \mathcal{T}_h is of weakly acute type (every angle of every $\tau \in \mathcal{T}_h$ is less than or equal to $\pi/2$) then the matrix associated with element-wise exponential fitting is an M-matrix.
2. Prove that if \mathcal{T}_h is Delaunay then the matrix associated with edge-wise exponential fitting is an M-matrix.
3. Code different methods for the following 1-D convection-diffusion equation:

$$-\varepsilon u'' + u' = 1, \quad x \in (0, 1) \quad u(0) = u(1) = 0.$$

- uniform grids, standard finite element (or central difference);
- uniform grids, upwinding method;
- adaptive grids, standard finite element. The grid is obtained by equidistributing the monitor function $M = (1 + \varepsilon^{-2} e^{-x/\varepsilon})^{1/2}$. That is the grid $0 = x_0 < x_1 < \dots < x_N = 1$ satisfies

$$\int_{x_i}^{x_{i+1}} M dx = \frac{1}{N} \int_0^1 M dx.$$

- *(Optional) arbitrary grid, streamline diffusion method.

Test the convergent rate for each method.