INTRO GROUP THEORY (MATH 120 A)

Final Examination (sample)

Problem 1.

Find the commutator subgroup of D_4.

Problem 2.

Give an example of a non-trivial homomorphism from $\mathbb{Z}$ to S_3. Is it possible to construct a homomorphism $\varphi : \mathbb{Z} \to S_3$ such that $\varphi(\mathbb{Z}) = S_3$?

Problem 3.

Let X be a G-set. Show that G acts faithfully on X if and only if no two distinct elements of G have the same action on each element of X.

Problem 4.

How many $\sigma \in S_5$ are there with

a) $\sigma^2 = id$?

b) $\sigma^3 = id$?

Problem 5.

Let $\varphi : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be a homomorphism such that $\varphi(1, 1) = 2$, and $\phi(3, 5) = 6$. Find $\mathrm{Ker}\varphi$ and $\varphi(10, 5)$.

Problem 6.

Find the maximal possible order of some element of $\mathbb{Z}_6 \times \mathbb{Z}_{36} \times \mathbb{Z}_{10}$.

Problem 7.

Classify the group

$$(\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z})/ < (1, 2, 3) >$$

according to the fundamental theorem of finitely generated abelian groups.

Problem 8.

Let H be a normal subgroup of a group G. Show that the center $\mathcal{Z}(H)$ is also a normal subgroup of G.