MATH EXAM ID#________________________

Analysis Comprehensive Exam
Spring 2017
June 20, 2017

All problems worth 10 points.

Question	Score
1	
2	
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Problem 1: Recall that a metric space X is *pathconnected* if, for any two points $x, y \in X$, there is a continuous function $\gamma : [0,1] \to X$ such that $\gamma(0) = x$ and $\gamma(1) = y$. We say that X is *locally pathconnected* if every point in X has a pathconnected neighborhood. Prove that if X is connected and locally pathconnected, then X is actually pathconnected.

Problem 2: Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) := \sum_{n=2}^{\infty} \left(\frac{x}{\ln n} \right)^n .$$

Is f continuous on $\mathbb{R}$? Justify your answer.

Problem 3: Suppose that E is an open subset of $\mathbb{R}^2$ and that $f : E \to \mathbb{R}$ is a function such that $D_1 f$, $D_2 f$, and $D_{21} f$ exist on E. Further suppose that $D_{21} f$ is continuous at $(a, b) \in E$. Prove that $D_{12} f(a, b)$ exists and $D_{12} f(a, b) = D_{21} f(a, b)$.

Problem 4: Let $h : \mathbb{R} \to \mathbb{R}$ be a bounded function and define $F : \mathbb{R} \to \mathbb{R}$ by

$$F(x) := \int_{-\infty}^{x} \sin(u^2) e^u h(u) du.$$

Prove that F is continuous.

Problem 5: Suppose that X is a compact metric space and that, for each $n \in \mathbb{N}$, $f_n : X \to \mathbb{R}$ is a continuous function. Further suppose that $f_n(x) \leq f_{n+1}(x)$ for all $x \in X$ and all $n \in \mathbb{N}$ and that the sequence (f_n) is uniformly bounded. Prove that the sequence (f_n) converges uniformly to a function $f : X \to \mathbb{R}$.

Problem 6: Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a continuous function. Prove that

$$\int_0^x f(u)(x-u)du = \int_0^x \int_0^u f(t)dtdu.$$

Problem 7:

(a) Let S^2 denote the unit sphere in $\mathbb{R}^3$. Compute the boundary ∂S^2 of S^2.

(b) Suppose that ω is an exact 2-form in $\mathbb{R}^3$. Prove that $\int_{S^2} \omega = 0$.

Problem 8: Suppose that $f : [a, b] \to \mathbb{R}$ is an increasing function. Prove that f is integrable on $[a, b]$.