Print Your Name: ______________ ________________
last first

Print Your I.D. Number: __________ ______________

Complex Qualifying Examination
Time: 1:00 pm–3:30pm, September 15, 2016
Room: MSTB 110

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Notation. Let $D(z_0, R)$ denote the disc in $\mathbf{C}$ centered at z_0 with radius R.

1. Evaluate the following integral

$$\int_0^\infty \frac{x}{(1+x^5)}dx.$$

2. Suppose f is analytic in the annulus $A = \{z \in \mathbf{C} : 1/2 < |z| < 2\}$, and there exists a sequence of polynomials p_n converging to f uniformly on the unit circle $|z| = 1$. Show that f can be extended to be an analytic function on the disc $D(0, 2)$.

3. Prove or disprove there is a non-zero holomorphic function f in the complex plane $\mathbf{C}$ such that

$$|f(z)|^2 \leq |\cos z|, \quad z \in \mathbf{C}.$$

4. Let f be meromorphic in the complex plane $\mathbf{C}$ such that

$$|f(z)| = 1 \quad \text{on} \quad |z| = 1.$$

Prove f is a rational function.

5. Find the radius of convergence for

$$\sin\left(\frac{2}{(z-2i+2)(z-3+i)}\right) = \sum_{n=0}^{\infty} a_n z^n$$

6. Prove or disprove there is a holomorphic function f on $\mathbf{C} \setminus D(0,3)$ such that

$$f'(z) = \frac{z^2+1}{z(z-1)(z-2)}$$

7. Let f be analytic on the upper-half plane and satisfy $|f(z)| < 1$. Furthermore suppose $f(2+i) = 0$. Give an upper bound for $|f'(2+i)|$ and state which functions realize this extrema.

8. Let u be a real-valued harmonic function in $\overline{D(0,1)} \backslash \{0\}$ such that

$$\lim_{z \to 0} \frac{u(z)}{\log |z|} = 0.$$

Show that there is a harmonic function U on $D(0,1)$ such that $u(z) = U(z)$ for all $z \in D(0,1) \setminus \{0\}$.